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Final Technical Report

Structural Integrity of Aging Airplanes

Submitted to

Transportation Systems Center
U.S. Department of Transportation
Kendall Square
Cambridge, MA 02138

(Ga. Tech Project No: E-20-671)

Submitted by

Satya N. Atluri
Regents' Professor & Director
Computational Mechanics Center
Georgia Institute of Technology
Atlanta, GA 30332-0356

Tele: (404)894-2758
Fax: (404)894-2299

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By
Satya N. Atluri
Regents' Professor & Director
Center for Computational Mechanics
Georgia Institute of Technology, Atlanta, GA
30332-0356
Tele: (404)894-2758
Fax: (404)894-2299

Analytical Modeling of Fuselage Structures

1 Introduction

In the following, the results of a review of recent literature that has a bearing on the analysis of structural integrity of semi-monocoque structures are given. A list of key ingredients necessary for an accurate mathematical modeling is given. A set of specific recommendations, involving computational strategies for the development of state-of-the-science computer software for stress and fracture analysis of stiffened fuselage structures, is given.

2 Background

Most of the prior structural integrity analyses of fuselage panels treated the panels as stiffened flat plates. However, some recent studies [notably, by Ansell (1988), and Riks, Brogan, and Rankin (1988)] have pointed to the need for accounting for the effects of curvature of the fuselage panels, as well as the effect of transverse bulging of a crack in the skin. However, thorough analyses, that are rigorous from both mathematical and mechanics points of view, of these effects, are still lacking.

The problem of multiple-site-damage in a fuselage structure is highly nonlinear, because of: (i) the locally large deformations that result in crack bulging and 'pillowing' effects; (ii) the significant role that finite deformations are likely to play in the phenomenon of multiple-site-damage ahead of a dominant lead crack. A thorough assessment of these nonlinear effects is currently lacking.

The multiple-site-damage in a fuselage skin, usually made of high toughness aluminum alloys, is likely to involve large-scale-plasticity near the crack-tips. In the presence of such large-scale-yielding, the concept of a linear elastic stress intensity factor to characterize the crack-tip field is highly vacuous. Also, the concept of energy-release-rate as employed in Riks, Brogan, and Rankin (1988) is not valid. Thus, a valid methodology for fracture and fatigue analyses in the presence of large scale yielding is needed to assess multiple-site-damage. The key ingredients in such a methodology are: (i) the use of valid parameters such as T^* [see the monograph by Atluri (1986) for a detailed discussion] in the presence of arbitrary loading/unloading histories and large-scale-plasticity; (ii) effective schemes to compute/estimate such crack-tip parameters, especially in the case of MSD, and (iii) *establishment and validation of criteria* for the assessment of the stability of growth of a crack/or cracks in the case of *multiple, interacting cracks*, each with a large-scale-yielding near its tip.

Against this background, the following report, with its literature review, and specific recommendations, is written.

3 Key Ingredients of, and Specific Recommendations for, State-Of-The-Science Computational Software for Modeling and Simulation of MSD in Fuselage Structures

3.1 Alternating Method for the Analysis of Multiple, Interacting Surface Flaws Emanating from Rivet-Holes in Fuselage Structures

A comprehensive and thorough review of computational methods for three-dimensional problems of fracture has been provided by Atluri and Nishioka (1986), and updated in Atluri and Nishioka (1989). It has been shown in these references [see also the monograph, Atluri (1986)] that, of all the variety of the feasible computational techniques, the Schwarz-Neumann alternating method provides the most cost-effective, yet the most accurate, means of evaluating the K-factors along the fronts of surface flaws such as quarter-elliptical or semi-elliptical surface flaws emanating from rivet holes in *curved* fuselage panels. This is because:

·The method models only the uncracked solid with finite elements or boundary elements; hence, no special modeling of the crack front is required. In addition, the finite element or boundary element mesh at the location of the crack, in the uncracked solid, can be completely arbitrary in geometry.

·The method uses the closed-form solution for a crack in an infinite solid which can accommadate arbitray tractions on the crack surfaces and, therefore, can handle complex loading conditions. Also, no *ad hoc* assumptions regarding the crack-tip constraint ("plane-stress" or "plane-strain") is necessarily made. This closed form solution has already been derived by Vijayakumar & Atluri (1981) and Nishioka and Atluri (1984).

·The stress intensity factors, including the individual modes, are obtained as a part of the solution, in an analytical form, and, hence, post-processing of the output data, as is usually done in the finite-element or boundary-element method, is not needed.

·The method may be combined with the influence function technique, or the weight function method, and thus provides important information for complete stress-intensity variations, or the weighted-average and/or local stress intensity factors, for crack-growth-rate evaluations and life predictions.

·Several crack configurations can be analysed with a single arbitrary mesh idealization of the uncracked solid, whereas the conventional finite-element or boundary-element method requires a different mesh idealization of the cracked structure for each crack configuration. Thus, this method can efficiently generate very accurate stress-intensity factors, weight functions or influence functions, for a variety of crack aspect ratios, in a single computer run.

Exploratory studies of the effects of interaction of the stress-fields due to *multiple surface flaws*, on the stress-intensity factor variation of each flaw, have been conducted by O'Donoghue, Nishioka, and Atluri (1984, 1985), and O'Donoghue, Nishioka, and Atluri (1986).

At the moment, this alternating procedure is embedded in only research-oriented computational software at the Computational Mechanics Center at Georgia Tech. It is recommended that this procedure be embedded in more production-oriented finite-element or boundary-element software, that is also currently available at the CMC at Georgia Tech. This task involves not only computer-code enhancement, but also additional developmental work.

3.2 Influence Functions and Weight Functions for Interacting Multiple Flaws in Fuselage Structures

Influence functions for surface flaws are the stress-intensity factor variations corresponding to a given (polynomial) type of traction distribution on the crack-surface, in an otherwise uncracked structure. Thus, if the tractions at the location of the carck, in an otherwise uncracked structure, are known, these tractions have to be released to create the given crack. By decomposing the thus given tractions into their polynomial type components, the stress-intensity factors for the given tractions can be easily computed from the known influence functions, through superposition. Thus, once the influence functions are stored in a computer database, repeated computational analyses of the flawed structure become unnecessary.

On the other hand, weight functions are the normalized variations of the crack-surface displacement fields due to variations in the crack-front profiles. Once these weight functions are evaluated for a given crack, under a reference load condidtion, the stress-intensity factor for *any other load condition* can simply be evaluated by using work-like integrals of the weight functions multiplied by the crack-face-tractions corresponding to the given load-condition. Thus, once the weight-functions are stored in a computer database, the evaluation of fatigue growth of surface flaws, such as those near rivet holes, becomes a simple task, without ever resorting to finite element or boundary element type computational models.

Recently, some fundamental break-throughs have been achieved in obtaining *analytical* expressions for weight functions for elliptical, circular, part-elliptical, and part-circular three-dimensional flaws, in typical structural geometries, by Nishioka and Atluri (1989) and Liao and Atluri (1989). However, the work so far has been limited to the case of a single flaw in a structure.

It is recommended that the alternating technique be used to develop influence functions and weight functions for a variety of crack aspect-ratios, and for typical situations of *multiple* interacting surface flaws emanating from rivet holes.

3.3 Elastic and Elastic-Plastic Analysis of Multiple Through-the-Thickness Flaws in Curved Fuselage Panels

Once the surface-flaw near a rivet hole grows, by fatigue, through the thickness of the fuselage skin, one has the situation of multiple through-thickness cracks emanating from a row of rivet holes in the fuselage. These through-thickness cracks are subjected to the combined tension-bending stress-fields of the curved fuselage skin.

One has to analyse the elastic or elastic-plastic asymptotic fields near the fronts (through the skin-thickness) of these interacting multiple flaws, to analyse the interactive growth and stability of such flaws.

Once again, the alternating method provides the simplest, and yet the most accurate, means of analyzing the interactive effects of these multiple through-thickness cracks. Exploratory studies of such alternating procedures for multiple cracks, using finite element or boundary element methods to model the *uncracked structure* only, have been recently conducted by Chen and Atluri (1989) and Rajiyah and Atluri (1989) for *flat panels*. It is recommended that such analysis be extended, and embedded in more user-oriented software, for *curved panels* containing rivet holes, stiffeners, etc.

While the Schwarz-Neumann alternating technique, in strict mathematical terms, is applicable only for fully linear problems, it may also be used for *elastic/plastic problems*, in the context of a valid engineering approximation. This is especially the case when a generalized mid-point-radial-return algorithm is used to predict plastic strains, and hence the stress redistribution, using the approximate total strains predicted from a *linear analysis*. Such radial-return algorithms have been comprehensively studied at the Computational Mechanics Center at Georgia Tech. In this scenario, a very coarse-grid finite element discretization of the curved fuselage panel, *without the multiple cracks*, will be first used. Then analytical linear elastic solutions for multiple interacting cracks will be used to erase the tractions at the locations of the cracks. In this process, approximate estimates of the total strain-rates will be made. Then, the mid-point-radial-return type constitutive-integrator algorithms will be used to predict *plastic-strains* and the resultant stress redistribution. Equilibrium-check type iterations will be used to check the correctness of the redistributed stresses. This iteration of the linear-alternating-technique subroutine will be carried out until a 'true' (converged) solution is obtained. The proposed solution methodology will result in a computational economy of at least two orders of magnitude, and is recommended to be developed as an engineering tool for the *elastic-plastic* analysis of interacting multiple through-thickness cracks in curved fuselage panels.

3.4 Effects of Curvature of the Panel, and Transverse Bulging, on a Through-Thickness Crack in a Fuselage

When a dominant flaw exists along the maeridional direction of the fuselage, and emanates, say, from a rivet hole, the pressure in the panel tends to induce transverse bulging near the crack-tip. Thus, the problem is highly nonlinear due to the presence of large-scale plasticity, and of finite deormations close to the crack-front. Furthermore, the problem is inherently three-dimensional in the vicinity of the crack-front, while it can be treated effectively as a two-dimensional case far away from the crack-front.

Thus, it is recommended to carry out a global/local analysis. First, a two-dimensional (shell-theory) type of analysis is to be carried out on the fuselage panels with stringers, ring-stiffeners, etc., using a *small deformation* theory. Using the boundary conditions provided by this two-dimensional analysis, a *three-dimensional elasto-plastic, finite-deformation* analysis of the immediate vicinity of the crack-front has to be carried out. This three-dimensional analysis will provide a much-needed understanding of the effect of bending-stress-fields on the crack front and its subsequent growth.

Computer software capable of carrying out the recommended global/local multi-scale analyses, that are available in skeletal froms at the Computational Mechanics Center at Georgia Tech, may be tailored for this specific task.

3.5 Appropriate Crack-Tip Parameters for Cracks in Fuselages, with the Nonlinear Effects of Large-Scale-Plasticity and Transverse Bulging Being Accounted For

It is well known that in the presence of large-scale plasticity near the crack-tip, the concept of an "energy-release-rate" to characterize the initiation and stability of crack-growth, is no longer valid [see the monograph by Atluri (1986) for a detailed discussion]. Despite this, energy-release-based criteria continue to be used [see, for instance, Riks, Brogan, and Rankin (1988)].

An appropriate crack-tip parameter in the presence of large-scale plasticity, finite deformations, and *arbitrary loading/unloading* as under fatigue, is the so-called T^* integral parameter [see the monograph by Atluri (1986) for a detailed justification].

It is defined as:

$$T^* = \int_{\Gamma_\epsilon} [W(F_{ij})n_1 - t_{ij}n_j \frac{\partial u_i}{\partial x_1}]ds \tag{1}$$

where, in the presence of *large-deformations* and plasticity;
Γ_ϵ = a contour,of a very small radius ϵ, near the crack front
W = stress-work density
$= \int_0^{F_{ij}} t_{ij} dF_{ij}$
F_{ij} = deformation gradient = $(\delta_{ij} + u_{i,j})$
t_{ij} = the unsymmetric first Piola stress
u_i = displacement
x_i = coordinates of a material particle in the *undeformed* configuration
n_j = direction cosines of a unit outward normal to Γ_ϵ in the undeformed configuration
$t_{ij} = JF_{ik}^{-1}\tau_{kj}$
τ_{kj} = Cauchy stress in finite deformation.

It is noted that T^*, as defined above for finite deformation elasto-plasticity, resembles the well-known definition of J as given by Rice (1968) for small-deformation elasticity. However, there are *fundamental* mathematical and physical differences between J and T^*. The Rice's J is usually defined as a far-field integral, which, for the elasto-static case involving material homogenity, is a path-independent *line integral.* However, for elastic-plastic materials undergoing arbitrary loading/unloading T^*, as a *contour integral alone*, is not path-independent. However, over arbitrary contours, T^* takes on the invariant expression,

$$T^* = \int_{\Gamma_\epsilon} [W(F_{ij})n_1 - n_j t_{ij} \frac{\partial u_i}{\partial x_1}]ds \tag{2}$$

$$\equiv \int_{\Gamma} [W(F_{ij})n_1 - n_j t_{ij} \frac{\partial u_i}{\partial x_1}]ds \tag{3}$$

$$- \int_{v_\Gamma - v_\epsilon} [\frac{\partial W}{\partial x_1} - \frac{\partial}{\partial x_j}(t_{ij}\frac{\partial u_i}{\partial x_1})]ds \tag{4}$$

Note that, for elastic-plastic materials, $\partial W/\partial x_1$ depends explicitly on x_1, since, within v_Γ, some material may undergo plastic loading, some may undergo unloading, and yet some other material is purely elastic. Thus $(\partial W/\partial x_1)$ needs to be computed explicitly.

It has been conclusively demonstrated by Brust and Atluri (1987) that T^* governs the growth and stability of elastic-plastic cracks under fatigue type loading/unloading cycles.

It is recommended that T^* be evaluated for cracks involving large-scale-yielding and transverse bulging.

An efficient computational algorithm for evaluating T^* involves an "equivalent domain integral" approximation. By introducing an arbitrary and continuous function G such that:

$$\mathrm{G} = 1 \quad at \quad \Gamma_\epsilon \tag{5}$$

$$= 0 \quad at \quad \Gamma, \tag{6}$$

one may develop a domain-independent-integral definition of T^* as:

$$T^* = -\int_{v_\Gamma - v_\epsilon} [W\frac{\partial G}{\partial x_1} - t_{ij}\frac{\partial u_i}{\partial x_1}\frac{\partial G}{\partial x_j}]ds \tag{7}$$

$$-\int_{v_\Gamma - v_\epsilon} [\frac{\partial W}{\partial x_1} - \frac{\partial}{\partial x_j}(t_{ij}\frac{\partial u_i}{\partial x_1})]Gds \tag{8}$$

for *arbitrary* v_Γ. The numerical algorithms for evaluating the above *domain integral*, with ease, may be patterned after the earlier work for small-deformation problems by Nikishkov and Atluri (1986, 1987).

It is recommended that a post-processing capability to evaluate T^* for finite plasticity, and arbitrary load-histories, based on the domain-integral approximation, be developed in the proposed computer software.

3.6 Advanced Theories for the Mechanics of Multiple-Site-Damage in Fuselage Panels: Stability of Interacting Multiple Elastic-Plastic Cracks

The growth patterns of a system of multiple cracks in an elastic-plastic panel may involve abrupt changes. One crack may suddenly grow to a large length, others may stop; and yet 2 or 3 cracks may grow and coalesce into a single dominant flaw. Thus, the concepts of a fundamental equilibrium path, bifurcation, and saddle points, are germane in the crack-growth regime. Thus, a *stability analysis of interacting* multiple flaws is necessary.

Suppose that T^* is used to characterize the crack-tip environment for each crack, of length $a_i (i = 1, ...n)$. Dimensionally, T^* is the first variation with respect to a_i of the total potential (stress-work minus the potential of surface tractions) of the panel. Using the well-known concepts of stability of equilibrium paths of deformable bodies, it appears natural to postulate that the stability of growth of one flaw, in a system of multiple flaws, is related to the second derivative of the total potential with respect to a_i. This concept is recommended to be explored.

First, an analysis of a set of model problems is recommended. For instance, one may consider a curved panel with two rivet holes and two cracks, each emanating from each of the holes. If one crack grows, the T^* of each crack may be computed, and thus, $\partial T_1^*/\partial a_1$ and $\partial T_2^*/\partial a_2$ may be computed. It may be postulated that the growth of either crack is stable, if $\partial T_1^*/\partial a_1$ and $\partial T_2^*/\partial a_2$ are < 0; unstable if $\partial T_1^*/\partial a_1$ and $\partial \partial T_2^*/\partial a_2$ are > 0; and the equilibrium is a critical state, if $\partial T_1^*/\partial a_1$ and $\partial T_2^*/\partial a_2$ are zero.

It is recommended that the above postulates be verified in carefully selected experimental/analytical programs. If such verifications are positive, the next step is to develop simple schemes for the *estimation* of T^* of each crack, without using elaborate finite element computations.

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